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Modelling Bose–Einstein correlations at LEP 2

Leif Lönnblad[1], Torbjörn Sjöstrand[2]

[1] NORDITA, Blegdamsvej 17, DK-2100 København Ø, Denmark (e-mail: leif@nordita.dk)
[2] Department of Theoretical Physics, Sölvegatan 14A, S-223 62 Lund, Sweden (e-mail: torbjorn@thep.lu.se)

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Abstract. We present new algorithms for simulating Bose–Einstein correlations among final-state bosons in an event generator. The algorithms are all based on introducing Bose–Einstein correlations as a shift of final-state momenta among identical bosons, and differ only in the way energy and momentum conservation is ensured. The benefits and shortcomings of this approach, that may be viewed as a local reweighting strategy, is compared to the ones of recently proposed algorithms involving global event reweighting. We use the new algorithms to improve on our previous study of the effects of Bose–Einstein correlations on the W mass measurement at LEP 2. The intrinsic uncertainty could be as high as 100 MeV but is probably reduced to the order of 30 MeV with realistic experimental reconstruction procedures.

1 Introduction

Most of the particles produced in hadronic events are pions, and as such they obey Bose statistics. One therefore expects an enhancement of the production of identical particles at small momentum separation, relative to what uncorrelated production would have lead to [1]. The shape of the enhancement curve reflects the size of the space–time region over which particle production occurs and the mechanism of particle production. Measurements of Bose–Einstein (BE) effects therefore directly test our understanding of QCD, in a way very much complementary to other QCD studies.

Unfortunately, the nice basic idea has complications. We do not have a solution of nonperturbative QCD even for the case of nonidentical particles, let alone for identical ones. Thus we do not know how to write down the amplitudes that, when symmetrized, should lead to a BE enhancement. That is, theoretical studies have to be based on models, and so shortcomings in comparisons with data may be difficult to localize. From the experimental point of view, the extraction of an unbiased BE enhancement curve is impossible, since there is no access to an alternative world not obeying BE statistics but otherwise the same. Reference samples can be defined in various ways, but all suffer from limitations.

That notwithstanding, studies of multihadronic events show clear evidence of BE enhancements [2–4]. If the enhancement of the two-particle correlation is parametrized in the phenomenological form

$$f_2(Q) = 1 + \lambda \exp(-Q^2 R^2) \ , \qquad (1)$$

one finds $\lambda \sim 1$ and $R \sim 0.5$ fm in hadronic $\mathrm{e}^+\mathrm{e}^-$ annihilation events. Here Q is the relative difference in four-momenta, $Q^2 = Q_{12}^2 = -(p_1 - p_2)^2 = m_{12}^2 - 4m^2$. The $\lambda \sim 1$ value refers to production at the primary vertex; decays of long-lived resonances and other dilution effects lead to the observable values typically being more like 0.2–0.3. The R parameter does not have to have a simple interpretation, but can be identified with a source radius in geometrical models [5].

One interesting question is whether BE correlations only affect our understanding of QCD, or whether it has wider implications. In a previous publication [6] we investigated possible BE effects on the W-mass measurement at LEP 2. Such effects can be expected in the purely hadronic channel because the space–time regions of hadronization of the two W bosons are overlapping. Using an algorithm which models BE correlations in the Pythia [7] event generator in terms of a ‘final-state interaction’ between identical bosons, we found that the effects on the measured mass in the purely hadronic channel, also called the four-jet channel, m_{W}^{4j}, may be very large. Although the algorithm had some shortcomings, it was the first serious attempt to estimate this effect and still represents a thought-provoking ‘worst case’ scenario indicating a systematic uncertainty of more than 100 MeV on m_{W}^{4j}.

Since our first publication, several other studies have been performed [8–12], giving small or vanishing effects on m_{W}^{4j}. Contrary to our approach, these new algorithms are mainly based on a global reweighting of events to obtain the observed correlations between identical bosons. It is often argued that such algorithms are more ‘theoretically appealing’ than the local reweighting perspective that is implicit in our momentum shifting strategy. As we point out in [6] and also stress in this paper, this need not be the case: the global reweighting philosophy can give unexpected and unphysical side effect. We cannot therefore today claim that there is one ‘best’ recipe. As long as these uncertainties persist, we cannot exclude a significant systematic shift on m_{W}^{4j}.

It may, however, be possible to use other experimental observables than m_{W}^{4j} to rule out one or several models. One such observable is presented by DELPHI [13]. By a clever combination of semi-leptonic and fully hadronic events, they can isolate the BE effects due to correlations between pions from different W bosons. The statistics is rather small, and so does not really discriminate between models, but it is still interesting that DELPHI finds no trace of such BE effects. Recently ALEPH came to the same conclusion [14]. Should these results survive an increase in statistics, it would require a revision of our current understanding of such BE effects and would surely rule out a significant shift of m_{W}^{4j} by this source. It would favour a scenario where the W^+ and W^- systems appear as uncorrelated sources of particle production, in spite of their space–time overlap. While the (lack of) BE enhancement does not directly probe other possible sources of mass shifts, such as colour rearrangement [16,17], a null result would make it plausible that also these other sources are negligible. From J/ψ production in B meson decay we know that the colour rearrangement mechanism does exist, however, so conclusions have to be drawn with care.

The main problem with the the algorithm we presented in [6] is that energy conservation is explicitly broken in the treatment of individual particle pairs, and is restored only by a global rescaling of all final-state hadron momenta. This rescaling introduces an artificial negative shift in m_{W}^{4j}, and a rather cumbersome correction scheme is needed to unfold the positive shift due to BE effects. Therefore it was not feasible to study the consequences of realistic experimental reconstruction procedures. In this paper we present four new algorithms, all variations of the same basic 'final-state-interaction' approach, where not only momentum but also energy conservation is handled locally. The algorithms are presented in detail in Sect. 3. Before that, however, we have a discussion in Sect. 2 on the understanding and modelling of the BE phenomenon in general, to clarify some of the conceptual issues, in particular the reasons for us to pick a local approach to the BE phenomenon. In Sect. 4 we present some results using our new algorithms, and finally, we present our conclusions in Sect. 5.

2 Models and data for the BE phenomenon

As already emphasized in the introduction, we do not know how to include the BE phenomenon in descriptions of hadron production in high-energy interactions. In this sense, whatever is currently done has the character of 'cookbook' recipes, and should be taken with a pinch of salt. This does not mean that all approaches have to be put on an equal footing: the level of sophistication and the measure of internal consistency can easily vary between models.

2.1 Global vs. local BE weights

A possible characterization of models is in terms of 'global' and 'local'. In global models a BE weight W_{BE} can be associated with each individual event. More precisely, it is assumed that a model exists for particle production in the absence of Bose statistics, that can be used to draw an unbiased sample of events. In order to include BE effects, each such unbiased event obtains a weight that is the ratio of the squared matrix elements of the production process with and without BE, respectively. The art is then to derive as plausible matrix elements as possible, so that the ratio can be evaluated with some confidence. The hope is that a lot of our ignorance should divide out in the ratio, so that we do not need absolute knowledge of nonperturbative QCD to make some realistic predictions for W_{BE}.

The word 'global' is used to denote the character of the weighting procedure, in the sense that one weight is assigned to the event as a whole, rather than to a specific particle pair. The terminology is not intended to reflect the character of the BE phenomenon as such, which normally is assumed to be local in $(\Delta x, \Delta p)$ space. Thus the global weight is typically built up as the product or sum of factors/terms that each by itself is of local character. The introduction of a global weight still leaves the door open for intentional or spurious BE effects of a non-local character; e.g., the strength of the BE enhancement in one region of an event could be influenced by the total multiplicity in the rest of the event.

A global weight can be given different interpretations. Often it is viewed as a multiplicative factor affecting the production rate of a given final state. In such approaches, there are some well-established experimental facts that have to be taken into consideration. Main among those is that the width of the Z^0 resonance agrees extremely well with the perturbative predictions of the standard model [15]. If indeed there is a global BE weight W_{BE} for each event, such that

$$\Gamma_{\mathrm{Z}}^{\mathrm{total}} = \Gamma_{Z}^{\mathrm{leptonic}} + \Gamma_{\mathrm{Z}}^{\mathrm{invisible}(\nu)} + \Gamma_{\mathrm{Z}}^{\mathrm{hadronic,perturbative}} \cdot \langle W_{\mathrm{BE}} \rangle \qquad (2)$$

then $\langle W_{\mathrm{BE}} \rangle = 1$ to a precision much better than 1%. This immediately excludes models where weights always are above unity, since a reweighting of events only at the per cent level could not explain the order unity BE enhancements in the data.

Although precision is highest for Γ_{Z}, some other related conclusions can be drawn from other data. The $\langle W_{\mathrm{BE}} \rangle$ cannot be a function of energy, since $R = \sigma(\mathrm{e}^+\mathrm{e}^- \to \mathrm{hadrons})/\sigma(\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-)$ agrees with perturbative predictions over a wide range of energies. It also cannot be a function of initial quark flavour, since the b quark fraction of Z^0 decays agrees with electroweak theory. It appears implausible that BE weights could change the relative composition of partonic states, since both the distribution in number of jets and in angles between jets agree very well with perturbative QCD predictions, also when based on an α_{s} determined from other processes. In passing, we note that BE effects among the perturbative gluons are significantly reduced by the existence of eight different colour states and are expected to be negligible.

Finally, the hadronic multiplicity varies as a function of energy and primary flavour, so the weight cannot be a function of the multiplicity in a direct way. Implicitly it would still be, of course, in the sense that a larger multiplicity for fixed energy and flavour means particles are packed closer in phase space on the average, i.e. pairs have lower Q values. The increase of the average multiplicity with energy could then be viewed as reflecting an increase in the phase space available for particle production, with unchanged average particle density in this phase space [18].

As we shall see, several models based on global weights have difficulties in accommodating these experimental observations. From a theoretical point of view, all the observations are naturally explained by them having a common origin in the factorization property of QCD [19]. Simply put, factorization tells that nonperturbative physics cannot influence the hard perturbative phase, or at least that any such corrections have to be suppressed by powers of $1/Q^2$, where Q here denotes the energy scale of the perturbative process. This may be viewed as a natural consequence of the time-ordering of the process, where first the Z^0 decays to a $q\overline{q}$ pair, which then may emit further partons that stretch confining colour fields, strings [20], between themselves. The hadron production from the string pieces only occurs at time scales of a few fermi in the center of the event, and even later for the faster particles. By this time it is 'too late' to influence the original selection of q flavour or (early) partonic cascade, but instead the hadronization process is likely to proceed with unit probability to some final state.

Whereas many models with global weights break factorization, the ones with local weights take factorization as their starting point. A parton configuration, once given by the perturbative rules, is fixed. Any weighting that enhances some fragmentation histories must, in exact balance, deplete others with the same parton configuration. Furthermore, the $R \sim 0.5$ fm value indicates that the BE effect occurs predominantly on a local scale, affecting particles that are produced fairly nearby along the string. Therefore, in the local models, it is assumed that the hadronization at one end of the string occurs (almost) independently of that at the other end. This is already part of the standard string fragmentation approach, without BE, as a natural consequence of causality. The acausality effects of the BE phenomenon are assumed to spread over distances of the order of R, in reality maybe some few fm, but still small compared with the total size of the fragmenting system at LEP energies. It is therefore assumed meaningless to define a weight that attempts to bring together information about widely separated parts of the event. Instead the local weight strategy is based on applying a reweighting procedure for each pair of identical particles in a way that only affects the local neighbourhood of the pair. In practice, the BE phenomenon becomes reduced to a kind of final-state interaction: the BE reweighting is a modest perturbation on events that, by and large, are given by the no-BE scenario. This does not have to mean that underlying physics is that of a final-state interaction, only that the algorithms for local weights can be made more tractable when reformulated in those terms. Specifically, events generated without BE effects can be perturbed, by shifts in the momenta of the particles, in such a way as to give the desired two-particle correlations [21,6]. This procedure can be applied event by event, with unit probability.

It should be clear to the reader that we lean towards the local weigh approach rather than the global weight one, since we do take the experimental data and theoretical dogma of factorization seriously. However, having said that, it must be admitted that the principles of local weights does leave room for alternative and arbitrary choices, e.g. as to how energy and momentum is conserved locally. It is this arbitrariness that will be studied in the subsequent sections. The global weight approach does not have the corresponding problem, since the reweighting is automatically between configurations that all have the same energy and momentum. Currently the choice is therefore between the global models, that have a more appealing implementation but often contradict our current understanding of QCD, and the local ones, that have a more sound basis in the factorization properties of QCD but lead to rather ugly technical tricks. The distance between the ideal model and the algorithms actually used may therefore be larger in the local approach. Specifically, what is studied in this paper is a set of local algorithms rather than the local concept as such.

It is possible to construct models intermediate to the pure 'global' and 'local' extremes. In one existing model [11] factorization is ensured by always retaining a parton configuration, once it has been selected according to the perturbative rules. Only the subsequent hadronization step is assigned a weight, and repeated until accepted by standard Monte Carlo procedure. Also BE effects in decays are considered separately from the main reweighting loop. Thus the global weight aspects are minimized.

2.2 Multiplicities

A measure of our ignorance of the BE phenomenon is that we do not know whether it is supposed to change the multiplicity distribution of events or not. That is, does the 'BE bump' at small momentum separation Q values correspond to an extra number of particles in the event, that would not have been there in a world without Bose statistics? In thermal field theory one can prove that $f_2(Q) \geq 1$ everywhere [22], which would indicate that BE indeed does increase the average multiplicity, or at least changes the multiplicity distribution to favour the high-multiplicity tail. However, the field theoretical definition of $f_2(Q)$ cannot be directly applied to e^+e^- events, so already for this reason it is difficult to draw any conclusions. Furthermore, one of the necessary assumptions is that extra particles can be produced at no cost in energy/momentum/charge/flavour conservation. This may be a sensible approximation for the central rapidity region of heavy-ion collisions at very high energies (and even so it turns out to be problematical to implement BE models [23]), but has little to do with our understanding of physics in e^+e^- annihilation. Rather, a model like the string one implies that particle production is based on local flavour conservation, so that e.g. two positively charged particles could not appear as nearest neighbours in rank. The string tension of 1 GeV/fm also sets the scale for how closely particles can be produced. There is therefore no logical need to assume a BE change of multiplicity. Just like ordinary fragmentation contains multiplicity fluctuations, however, one could imagine that the BE mechanism favours the fluctuations towards higher multiplicities; this is particularly compelling in scenarios with global BE weights always above unity.

The data does not settle the issue. As conventionally presented, the BE enhancement at small Q is compensated by a dip of $C_2(Q)$ below unity at intermediate Q. (In the following, we use $C_2(Q)$ for the measured two-particle correlation and $f_2(Q)$ for the theory input.) This behaviour is well 'predicted' in our momentum shift algorithm, i.e. it involves no free parameters but comes from the formalism. In this sense, there is no case for a multiplicity change. However, experimental analyses are normally based on a reference sample for the imagined no-BE world picked to have the same multiplicity as the data. By definition, one thus assumes no multiplicity change, and the dip at intermediate Q is a logical consequence of this assumption. In model-independent fits, it is necessary to include a factor like $N(1 + kQ)$ (with $k > 0$ and $N < 1$), in addition to the form of (1), to describe the data. Such a factor has no simple interpretation in formalisms based on global weights always above unity. However, if one plays with the main 'b' parameter of the Lund longitudinal fragmentation function [20] to create a Monte Carlo no-BE reference world with a lower-than-real multiplicity, the need for the $N(1 + kQ)$ factor vanishes for a multiplicity ∼12% lower than the data [24]. The $C_2(Q)$ still drops below unity at very large Q, but this is an inevitable consequence of energy conservation and not in contradiction with weights always above unity. Finally, models with global weights both above and below unity can explain the experimental dip at intermediate Q as part of the weight variation but, depending on the details of the weight distribution, could additionally need to invoke some global multiplicity change. Any answer between 0 and ∼12% multiplicity change thus seems perfectly feasible to accommodate from an experimental point of view, depending on the model used to interpret the data.

One should also note what is *not* found in the data. The BE effect, especially for BE weights assumed everywhere above unity, could be expected to lead to 'runaway' situations where an event or a region of an event consists almost entirely of π^0's or $\pi^\pm$'s, since this would maximize the event weight. No signals for larger-than-expected fluctuations of this nature have been found in the data, indicating that the no-BE picture of uncorrelated flavour production at adjacent string breakup vertices (modulo some technical complications included in realistic event generators) is a good first approximation. However, we would welcome further studies, to quantify how big such effects could still be allowed by the data.

A perfectly plausible scenario is thus that BE effects do not change the particle number or composition of events, but only relative momentum separation between particles. This is the assumption pursued in our local scenarios.

2.3 Local approaches

Above we have argued for a local scenario, wherein all the major properties of the event can be given without any reference to the BE phenomenon. The BE effect is then introduced as a perturbation. This gives a large formal similarity with final-state interactions, although the underlying physics may well be different. Anyway, this similarity allows for a more tractable approach to the simulation of BE effects.

The algorithm presented in [6] takes the hadrons produced by the string fragmentation in Jetset, where no BE effects are present, and shifts slightly the momenta of mesons so that the inclusive distribution of the relative separation Q of identical pairs is enhanced by a factor $f_2(Q)$, e.g. of the form of (1). Making the ansatz that the original distribution in Q is just given by phase space, $d^3p/E \propto Q^2 dQ^2/\sqrt{Q^2 + 4m^2}$, an appropriate shift δQ for a given pair with separation Q can be given by

$$\int_0^Q \frac{q^2 dq}{\sqrt{q^2 + 4m^2}} = \int_0^{Q+\delta Q} f_2(Q) \frac{q^2 dq}{\sqrt{q^2 + 4m^2}}. \quad (3)$$

For an arbitrary $f_2(Q) \geq 1$, δQ is negative and pairs are pulled closer together. The pair density does not increase as fast as phase space implies once Q is larger than the typical transverse momentum spread of the string fragmentation. This leads to the generated $C_2(Q)$ dropping below unity at intermediate Q and approaching unity from below for large Q, see [6] for details. The choice of not using the actual phase space density is a deliberate one; we believe that the deviations from a pure phase space distribution of particles and the assumption of a conserved total multiplicity should have repercussions in terms of the output $C_2(Q)$ not agreeing with the input $f_2(Q)$.

The translation of δQ into a change in particle momenta is not unique. Since the invariant mass of a pair is changed, it is not possible to simultaneously conserve both energy and momentum, and so compromises are necessary. We have chosen to conserve three-momentum in the frame where the algorithm is applied. For a given pair of particles i and j the change is $\mathbf{p}'_i = \mathbf{p}_i + \delta\mathbf{p}_i^j$, $\mathbf{p}'_j = \mathbf{p}_j + \delta\mathbf{p}_j^i$, with $\delta\mathbf{p}_i^j + \delta\mathbf{p}_j^i = \mathbf{0}$, and we simply take $\delta\mathbf{p}_i^j = c(\mathbf{p}_j - \mathbf{p}_i)$ corresponding to pulling the particles closer along the line connecting them in the current frame. In [6] we also tried other strategies, such as conserving energy rather than momentum, and shifting the momenta of a pair in their rest frame, but we found that our results were not very sensitive to such choices.

A given particle is likely to belong to several pairs. If the momentum shifts above are carried out in some specific order, the end result will depend on this order. Instead all pairwise shifts are evaluated on the basis of the original momentum configuration, and only afterwards is each momentum $\mathbf{p}_i$ shifted to $\mathbf{p}'_i = \mathbf{p}_i + \sum_{j\neq i} \delta\mathbf{p}_i^j$. That is, the net shift is the composant of all potential shifts due to the complete configuration of identical particles. This means that the pair ansatz is strictly valid only for large source radii, when the BE-enhanced region in Q is small, so that the momentum shift of each particle receives contributions only from very few nearby identical particles. For normal-sized radii, $R \sim 0.5$ fm, the method introduces complex effects among triplets and higher multiplets of nearby identical particles, which (together with the phase space ansatz discussed above) is reflected both in changes between the input $f_2(Q)$ and the final output $C_2(Q)$ [6,25] and in the emergence of non-trivial higher-order correlations. The latter actually agree qualitatively with such data [26].

Short-lived resonances like ρ and K^* are allowed to decay before the BE procedure is applied, while more long-lived ones are not affected. This leads to a shift in the ρ^0 mass peak, something also observed in the data [27].

The above procedure preserves the total momentum, while the shift of particle pairs towards each other reduces the total energy. For a $\mathrm{Z}^0 \to \mathrm{q}\overline{\mathrm{q}}$ event this shift is typically a few hundred MeV, and so is small in relation to the Z^0 mass. In practice, the mismatch has been removed by a rescaling of all three-momenta by a common factor (very close to unity). As a consequence, also the Q values are changed by about the same small amount, whether the pairs are at low or at high momenta. That is, the local changes due to the energy conservation constraint have been minimized by spreading the corrections globally.

By and large, the very simple ansatz above gives an amazingly good account of BE phenomenology in $\mathrm{e}^+\mathrm{e}^-$ annihilation, including many genuine predictions. In addition to what has already been mentioned, one could note the variation of longitudinal, out and sideways fitted radii as a function of the transverse mass of a pair [30]. Some of these agreements may be coincidental, or trivial consequences of any reasonable BE implementation, but at least $\mathrm{e}^+\mathrm{e}^-$ data so far has not revealed any basic flaw in the simple original version of the local approach.

By contrast, in $\mathrm{p}\overline{\mathrm{p}}$ data the UA1 and E735 collaborations have observed that the λ parameter decreases and the R parameter increases with increasing particle density [28]. Neither behaviour follows naturally from our approach, although it could be argued that final-state interactions at least would be consistent with an increasing radius of 'decoupling' for larger multiplicities. Above we have attempted to explain our momentum-shifting strategy as being motivated more by a local reweighting philosophy than a final-state interaction one, in order to highlight similarities and differences with global weight schemes. In view of the $\mathrm{p}\overline{\mathrm{p}}$ data it might be prudent not to close the door on both effects being present in the data, and hopefully both being approximated by our algorithm.

The agreement with $\mathrm{e}^+\mathrm{e}^-$ data does not mean that the method is free of objections [3,29]. The deterministic nature of the momentum shift algorithm does not go well with the basic quantum mechanical nature of the problem, and is likely to mean that a potential source of event-to-event fluctuations is lost. The selected input form of $f_2(Q)$, like in (1), is not coming from any first principles, and λ and R are two free parameters. It could be argued that $\lambda = 1$ is a natural value, and that a transverse BE radius $R \sim 0.5$ fm is about the transverse size of the string itself, but it is not at all clear why a similar Gaussian form and radius should apply for the longitudinal degree of freedom. This would require a detailed study and understanding of the microscopic history of the event (as is offered in some global models [31,32,11]). Possibly it would then turn out that the shape used is reasonable on the average, even when a poor approximation for the individual event. For instance, the space–time history of string fragmentation gives, on the average, a coordinate separation of two particle production vertices proportional to the momentum difference between the particles. The Q^2 factor of $f_2(Q)$ could then be reinterpreted as being $\Delta x \cdot \Delta p$, and the longitudinal R related to longitudinal fragmentation parameters. However, the relation $\Delta x \propto \Delta p$ suffers from large fluctuations in the actual string histories, that are now completely neglected.

Another set of possible complications comes from the assumption that the BE phenomenon is the same in quark and gluon jets, in spite of the more complicated space-time structure of particle production in the gluon jets, cf. the following model. Our local scheme is here based on the simplest possible picture and, as for several of the aspects covered above (spherical source, no input three-particle correlation, ...), one could imagine more complicated variants of the local ansatz.

2.4 Global approaches

Whereas the local approach to the BE phenomenon only has been developed by us, many global algorithms have been proposed. It would carry too far to describe all, but we here would like to comment on a few of them, with special emphasis on those that have been used to study the issue of a m_{W}^{4j} shift.

The probably most sophisticated global approach is the one originally proposed by Andersson and Hofmann [31] and further developed by Andersson and Ringnér [32]. Here the fragmentation process is associated with a matrix element

$$\mathcal{M} = \exp(i\kappa - b/2)A \ , \qquad (4)$$

where κ is the string tension, b is related to the breaking probability per unit area of the string and hence to the form of the fragmentation function, and A is the total space–time area spanned by the string before fragmenting. String histories with different areas can lead to the same final state — the simplest example being the permutation of the momenta of two identical particles — so nontrivial interference effects are obtained when the amplitudes are added. This can be reformulated in terms of an effective weight

$$W_{\mathrm{BE}} = 1 + \sum_{\mathcal{P}' \neq \mathcal{P}} \frac{\cos \kappa \, \Delta A}{\cosh\left(\frac{b\,\Delta A}{2} + \frac{\Delta(\sum p_{\perp \mathrm{q}}^2)}{2\sigma^2}\right)} \, , \quad (5)$$

where $\mathcal{P}' \neq \mathcal{P}$ indicates that the sum should run over all permutations of momenta of identical particles, except for the original configuration itself. The second term in the denominator comes from the transverse momentum degrees of freedom of quark pairs that have to have their transverse momenta reinterpreted by the permutation, and tends to dampen weights. The area difference ΔA between two string fragmentation histories is, for a simple pair permutation, equal to the product of the energy–momentum difference and the four-distance between the production points. The cosine in the weight numerator means that the $f_2(Q)$ distribution is expected to oscillate around unity, while the dampening of the weight denominator ensures that only the first peak and dip are visible in the end. The ρ and K^* decays are treated as if they were part of the string decay itself, so that the decay products can be symmetrized with primary particles. There are two technical complications: firstly, that an inclusion of all possible permutations would make the algorithm extremely slow and, secondly, that individual weights can be negative. The first point is ameliorated by a truncation, where only terms with a significant impact on results are retained. The latter point is an artifact of the algorithm and not a real problem.

The algorithm gives a good description of two-jet data, as far as it can be tested. However, it does give an average weight of about 1.2, that has to be divided out by hand. It is the oscillations of the weight function that gives it a value close to unity, with the actual number rather sensitive to fragmentation model parameters [33]. No clear physics interpretation is offered of the average weight, e.g. in the context of the Z^0 width. It has not been studied whether the algorithm gives a change in the jet number or primary flavour composition.

Technical complications means that the generalization of the model to three-jet events is less well studied. One consequence of the model is that a gluon jet is expected to contain less BE correlations than a quark one: the gluon fragmentation involves two string pieces, so that the distance between two particle production vertices, in absolute numbers or defined in terms of ΔA, is larger than implied by the momentum difference. In our local approach the full space–time hadronization history is not used, so this aspect is not caught. Therefore one obtains differences between models, although they may be difficult to observe [33].

The model of Todorova–Nová and Rameš [11] contains a global weight, but its importance is limited, so as to emphasize the local character of the BE phenomenon. In a first step, a parton configuration is selected according to conventional perturbative probabilities. In the second step, the partons are hadronized according to the string model, from which the production vertices of hadrons can be extracted. An event weight is given by

$$W_{\mathrm{BE}} = 1 + \sum_{\mathrm{all\ pairs}} \cos(\Delta x \cdot \Delta p)\, \theta\left(\frac{\pi}{2} - |\Delta x \cdot \Delta p|\right) \, , \quad (6)$$

where the cosine factor comes from wave function symmetrization and the θ step function ensures that only small $\Delta x \cdot \Delta p$ contribute. Also three-particle correlations are included in a similar spirit. Only primary π, K, ρ and ω particles, produced directly from the string, are included in the global weight. The number of primary particles of each species being rather small — e.g. about 16% of the charged pions are directly produced — the weight fluctuations are manageably small. The second step is iterated, i.e. the same parton configuration is re-hadronized, until the weighting procedure gives acceptance. This reweighting does shift the multiplicities of produced particles, but rather modestly. Particles from resonances (including short-lived ones like the ρ) are not part of the global weight. Instead, in the third step, decay kinematics is selected according to a probability distribution that follows the correlation function.

Kartvelishvili, Kvatadze and Møller have studied several models [9]. The most extreme is a global weight

$$W_{\mathrm{BE}} = \prod_{\mathrm{all\ pairs}} \left\{1 + \lambda \exp(-Q^2 R^2)\right\} \, , \quad (7)$$

which then gives an average weight much above unity, an increased average multiplicity (that can be tuned away), a much increased three-jet fraction and a reduced fraction of $\mathrm{Z}^0 \to \mathrm{b\overline{b}}$ decays. Since this is unacceptable, different rescaling schemes for the global weights are introduced. One is based on a suppression by a constant factor for each pair, another on normalizing to a weight also involving pairs of non-identical particles. Alternatively the pair weight in (7) is modified to $1 + \cos(\xi Q R)/\cosh(QR)$ with $\xi = 1.15$. These modifications reduce the problems noted above but do not solve them; additionally the rescalings are completely *ad hoc* and are given no physics explanation.

The model of Jadach and Zalewski [8] is based on a subdivision of the event into clusters of identical particles, to which a particle can belong only if it has a neighbour within a distance $Q < 0.2$ GeV. This cut is very visible in the final BE distribution, but is probably required to keep the clusters of tractable size. A weight, always above unity, is defined for each cluster, and a global event weight by the product of cluster weights. Since the multiplicity is increased by the reweighting, the weights are rescaled by a factor raised to the total pion multiplicity to bring the average multiplicity back. A further common factor is needed to bring the weights to an average of unity. Also the jet multiplicity then comes out about right, but issues such as the flavour composition in Z^0 decays have not been studied. The average multiplicity of a W pair is about 4% higher than the sum of two separate W's.

Fiałkowski and Wit employ a global weight that contains a sum of all possible permutations among identical particles. To retain a tractable number of terms to evaluate, the procedure is cut short at permutations involving at most five particles. Studies with cuts at lower values indicate that the procedure, at least for the inclusive BE distribution, should have converged by then. Weights are always above unity and tend to push up the multiplicity distribution. As above, a factor raised to the total pion multiplicity is used to restore the average multiplicity and another common factor applied to produce correct average weight. The possibilities of a change in the flavour composition of Z^0 decays or of the jet multiplicity have not been studied.

Several other algorithms based on global weights have also been proposed or studied recently [35]. Since these other models have not been used to study the issue of m_{W}^{4j}, and do not offer any unique insights in the interpretation of nonunit average global weights, we will not comment on them here.

2.5 The W mass determination

At LEP 2 the average space–time separation between the two W decays is less than 0.1 fm [16], to be compared with a typical BE radius of around 0.5 fm. When the W's decay to $\mathrm{q}\overline{\mathrm{q}}$ pairs, the quarks fly apart and stretch strings between themselves. These strings will overlap in the central region, whereas the outer parts will not in general. Only in the case that two partons from different W's travel out in almost the same direction does the overlap spread also to the outer regions, but most such events would not survive standard selection criteria, used to separate W pair events from backgrounds such as QCD 3-jets.

Any BE effects caused by the overlap between the W^+ and W^- hadronization systems should therefore predominantly occur among the centrally produced, low-momentum particles. In this region it may not be possible to speak about separate W^+ and W^- sources of particle production, but only about one single common source. Since the hadrons do not emerge tagged with their origin, the mass definition has to be based on an experimental clustering procedure, usually first into four jets and thereafter those paired to the two W's [36]. Possible biases in the detector and the procedure can be controlled by studying Monte Carlo events generated with the W^+ and W^- hadronization processes decoupled from each other. The shift in the outcome of the procedure when BE effects are included in full is then what we loosely refer to as a 'W mass shift'. This does not have to imply that the masses of the W propagators in the perturbative graphs are affected. Rather, the main point is that our limited understanding of the BE phenomenon reduces the ability to 'unfold' the hadronic data to arrive at the partonic picture.

In our standard local scenario [6] we found that a mass shift of around or even somewhat above 100 MeV could not be excluded. On the scale of the desired experimental accuracy of maybe 30 MeV [36], as required for precision tests of the standard model, this is a large number. However, put in the context of QCD physics in general, the uncertainty is not exceptional, neither on an absolute nor on a relative scale. Specifically, for effects related to nonperturbative physics, uncertainties of the order of a pion mass or of Λ_{QCD} are fairly common. We also found that the assumed 'attractive' form of the BE factor defined in (1) leads to an enhancement of production in the low-momentum region of large overlap between the W^+ and W^- sources, at the expense of somewhat faster particles. The result is that the W mass shift tends to be positive.

This kind of mass shift does not have to be unique for the momentum shift method used in our local approach, but could well arise also in global weight schemes. Just like in local algorithms, the outcome would depend on model details.

First of all, the BE phenomenon could affect the interpretation of the W propagators. To see this, it is convenient to start out from the QED case. The lowest-order process $\mathrm{e}^+\mathrm{e}^- \to \mathrm{W}^+\mathrm{W}^- \to \ell^+\nu_\ell\ell'^-\nu'_\ell$ contains two W masses that are perfectly defined by the momenta of the final leptons and neutrinos. If a photon is added to the final state, however, there are six charged particles that could have radiated it, including all possible interference contributions. The normal experimental procedure would be either to remove the photon altogether (relevent for initial-state radiation) or to add it to one of the W^+ and W^- systems. Clearly this is too coarse an approximation, in particular for photons well away from the collinear regions. So we lose the concept of a unique theoretical or experimental definition of the W masses of a given event. For the totally inclusive $\mathrm{W}^+\mathrm{W}^-$ cross section there is a general proof [37] that the radiative interconnection effects are suppressed by $O(\alpha_{\mathrm{em}}\Gamma_{\mathrm{W}}/m_{\mathrm{W}})$. The only exception is the Coulomb interaction between two slowly moving W's. By contrast, differential distributions could be distorted on the level of $O(\alpha_{\mathrm{em}})$. Only in the limit of vanishing W width would one expect to recover a unique theoretical separation of radiation. In QED it is always possible in principle to calculate the corrections necessary to extract the proper average W mass from a given experimental procedure. Since complete calculations have not been performed, however, some uncertainty may still remain [38].

For QCD there is no radiation from the initial state or the W's themselves, but only from the final quarks. Furthermore, colour conservation ensures that there are no interconnection effects to $O(\alpha_{\mathrm{s}})$. The totally inclusive $\mathrm{W}^+\mathrm{W}^-$ cross section is therefore protected to $O(\alpha_{\mathrm{s}}^2\Gamma_{\mathrm{W}}/m_{\mathrm{W}})$ [37]. Again differential distributions could contain larger effects, related to the inability to assign a gluon uniquely to either of the W^+ and W^- systems. This perturbative interconnection is suppressed by propagator effects for energetic gluons, as shown in [16]. In the soft region, where gluon energies are below the Γ_{W} scale, the propagator damping is not effective, and non-negligible effects cannot be excluded.

Extrapolating from this, it is not impossible that BE effects indeed have repercussions on the W propagator description. To the extent one could still speak about two different sources of particle production, an effect to a global weight would come e.g. from interchanging the production of two identical particles. That is, either pion no. 1 is produced by the W^+ and pion no. 2 by the W^-, or the other way around. Since the two pions have different momenta,

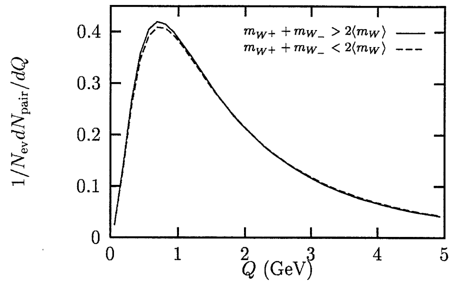

Fig. 1. The correlation function for pairs of pions with one pion from each W as a function of Q for two samples of $e^+e^- \to W^+W^-$ events at 170 GeV center of mass energy. The *full* (*dashed*) line corresponds to events where the average mass of the two W's is above (below) the nominal W-mass. Both curves are normalized to unity

in this case one would actually be considering interference between Feynman graphs with different W propagator masses. Each graph would have to be weighted with the respective perturbative production matrix elements, in addition to the BE weight. The exchange of two particles of widely different momenta is likely to push some W propagator off the mass shell and so suppress interference terms. For pairs within the BE enhancement region, however, the mass shifts will occur at a scale of a few hundred MeV, where the W propagator weight does not vary so drastically. The propagator effects are thus not expected to change the picture dramatically, but could well give some shift of the W mass. Since, to the best of our knowledge, none of the global models include the W propagators in their weights, this has not been put to a quantitative test. Furthermore the hadronization amplitude should be complex, cf. (4), as are the W propagators, something which could further complicate the interference pattern.

Another way a mass shift could arise in a global weight model is due to the fact that, for a given total energy, a heavy W will be less boosted away from the interaction point than a light one. This means that, for events with high-mass W's, the two fragmentation regions will have a larger overlap. A pair with one pion from each W is then more likely to be close to each other than in events with light-mass W's, as shown in Fig. 1. Events with heavier W's would thus be given a higher weight (provided the BE weight factor is always above unity), which could introduce a mass shift. Also, for a global weight model that does not conserve multiplicity, one would expect a higher weight for events with heavier W's, since the multiplicity increases with the mass.

In more complicated models, with a single source of particle production, the W mass concept would be questioned from the onset. However, we do not really know how to formulate such models, so all the ones studied to date are based on having a picture with two separate W's as starting point.

In the studies of Andersson and Ringnér the separation is an essential part of the model. The matrix element and weight expressions, (4) and (5), respectively, are based on a definition of the area spanned by each string. Therefore the weight of a pair of strings is the product of the weight of the respective string. If weights are rescaled to unity average for a string of any mass, it then follows by definition that the W mass is unaffected. It has also been shown [12] that effects are negligibly small, below $\sim$ 10 MeV, even when the weights are not rescaled. In this case a mass shift in principle could come from the variation of the average BE weight with the W mass, so the nonobservation of an effect can be reinterpreted in weight terms, but we remind that Z^0 and other data in principle exclude this use of nonunit average weights.

One should here recall the UA1 and E735 studies [28], which showed a decreasing λ parameter with increasing multiplicity density. This would arise quite naturally if large multiplicities were a consequence of having many strings in an event [39], with no BE cross-talk between strings. The simultaneous observation of an increasing BE radius R could be used to argue *for* the existence of cross-talk, however, so it may be premature to use UA1/E735 data as argument against a W mass shift.

The studies of Todorova–Nová and Rameš [11] also give a null result, within the statistical uncertainty of $\sim$ 10 MeV. This holds both for the average mass and a fitted mass peak value. Like in the previous model, the primary particle production factorizes into two sources by default. The 'theory' classification of particles into two groups would then still give unchanged masses. Several alternative scenarios were tried, checking for effects coming from misassigned particles and from a possible breaking of factorization, but none of them gives significant effects.

Kartvelishvili, Kvatadze and Møller do find a W mass shift with their methods [9], where the BE weight of an event is truly global, i.e. is not just the product of two separate weights but also contains cross-terms with one particle in a pair from each W. The shift in the average mass ranges between 20 and 75 MeV at 175 GeV and between 34 and 92 MeV at 192 GeV for the models studied. However, the authors note that the use of an average mass shift may be partly misleading, since typical experimental procedures are based on a fit to a central mass peak, so that the wings of the Breit-Wigners are suppressed in relative importance compared with a straight averaging. Within such a fitting procedure, the mass shift is still there but never larger than about 15 MeV, i.e. on an acceptable level.

Jadach and Zalewski, on the other hand, do not find a significant mass shift at all [8]: any possible signal is below the statistical error of 12 MeV. Again this is based on a fit to the mass peak. The model is reminiscent of one alternative studied by the previous authors, but uses a BE radius R of 1 fm rather than the 0.5 fm used there. Since the BE-affected phase space volume is reduced by an increased R, and since the cut $Q < 0.2$ GeV gives a further reduction, there does not appear to be any contradiction between these two studies [9].

Also Fiałkowski and Wit fail to find a significant mass shift, and quote a limit of 20 MeV [10]. Their Fig. 2 shows a very notable change of the shape of the W mass spectrum, however. The peak rate is reduced, while the rate in the wings is increased. This may indicate that the weight rescaling procedure is too simpleminded.

Even with the wings removed, the fitted W width is increased by 58 MeV when BE effects are included [40]. Since the fitting error is of the order of 30 MeV, the result would seem barely statistically significant. However, a visual inspection of their Fig. 2 leaves little doubt that the peak is broadened by BE, so the qualitative picture is not in question even if the exact number may be. If this broadening is another manifestation of weight rescaling imperfections then any results on the average W mass can hardly be trusted. If, on the other hand, it is a genuine consequence of the model, then it is in itself an even more interesting phenomenon than a shift of the peak position, and much simpler to study experimentally. Also the studies of Jadach and Zalewski give a fitted W width that increases with the inclusion of BE effects, by about the same amount as above [8]. Here, however, it is less easy to see from the curves in the paper whether this is a real phenomenon or just a fluke of the fitting procedure. For the other global models we have no information. More studies by the respective authors are here certainly called for, and below we report on results for our models.

In summary, we thus see that there is no unique answer. Many null results have been obtained, but also some nonzero ones. Some of the models may change the measurable W width even if the average W mass is unaffected. Obviously, to claim that the problem has 'gone away', it is not enough to find *one* method that give negligible mass or width shifts: one must find some reason to exclude *every* model that give uncomfortable values. We are not there yet. However, some of the criticism of our original study should be taken seriously, and below we study a few possible improvements.

3 New local algorithms

Probably the largest weakness of our local approach is the issue how to conserve the total four-momentum. The procedure described in Sect. 2.3 preserves three-momentum locally, but at the expense of not conserving energy. The subsequent rescaling of all momenta by a common factor (in the rest frame of the event) to restore energy conservation is purely *ad hoc*. For studies of a single Z^0 decay, it can plausibly be argued that such a rescaling does minimal harm. The same need not hold for a pair of resonances. Indeed, studies [6] show that this global rescaling scheme, which we will denote BE_0, introduces an artificial negative shift in m_W^{4j}, making it difficult (although doable) to study the true BE effects in this case. This is one reason to consider alternatives.

The global rescaling is also running counter to our original starting point that BE effects should be local. To be more specific, we assume that the energy density of the string is a fixed quantity. To the extent that a pair of particles have their four-momenta slightly shifted, the string should act as a 'commuting vessel', providing the difference to other particles produced in the same local region of the string. What this means in reality is still not completely specified, so further assumptions are necessary. In the following we discuss four possible algorithms, whereof the last two are based strictly on the local conservation aspect above, while the first two are attempting a slightly different twist to the locality concept. All are based on calculating an additional shift $\delta\mathbf{r}_k^l$ for some pairs of particles, where particles k and l need not be identical bosons. In the end each particle momentum will then be shifted to $\mathbf{p}'_i = \mathbf{p}_i + \sum_{j\neq i}\delta\mathbf{p}_i^j + \alpha\sum_{k\neq i}\delta\mathbf{r}_i^k$, with the parameter α adjusted separately for each event so that the total energy is conserved.

In the first approach we emulate the criticism of the global event weight methods with weights always above unity, as being intrinsically unstable. It appears more plausible that weights fluctuate above and below unity. For instance, the simple pair symmetrization weight is $1+\cos(\Delta x\cdot\Delta p)$, with the $1+\lambda\exp(-Q^2R^2)$ form only obtained after integration over a Gaussian source. Non-Gaussian sources give oscillatory behaviours, e.g. the conventional Kopylov–Podgoretskiĭ parametrization for particle production from a spherical surface [41]. The global model by Andersson, Hofmann and Ringnér is an example of weights above as well as below unity. In this case the oscillations contain the $\cos(\Delta x\cdot\Delta p)$ behaviour dampened by further factors at large values.

If weights above unity correspond to a shift of pairs towards smaller relative Q values, the below-unity weights instead give a shift towards larger Q. One therefore is lead to a picture where very nearby identical particles are shifted closer, those somewhat further are shifted apart, those even further yet again shifted closer, and so on. Probably the oscillations dampen out rather quickly, as indicated both by data and by the global model studies. We therefore simplify by simulating only the first peak and dip. Furthermore, to include the desired damping and to make contact with our normal generation algorithm (for simplicity), we retain the Gaussian form, but the standard $f_2(Q)=1+\lambda\exp(-Q^2R^2)$ is multiplied by a further factor $1+\alpha\lambda\exp(-Q^2R^2/9)$. The factor 1/9 in the exponential, i.e. a factor 3 difference in the Q variable, is consistent with data and also with what one might expect from a dampened cos form, but should be viewed more as a simple ansatz than having any deep meaning.

In the algorithm, which we denote BE_3, $\delta\mathbf{r}_i^j$ is then non-zero only for pairs of identical bosons, and is calculated in the same way as $\delta\mathbf{p}_i^j$, with the additional factor 1/9 in the exponential. As explained above, the $\delta\mathbf{r}_i^j$ shifts are then scaled by a common factor α that ensures total energy conservation. It turns out that the average α needed is ≈ -0.2. The negative sign is exactly what we want to ensure that $\delta\mathbf{r}_i^j$ corresponds to shifting a pair apart, while the order of α is consistent with the expected increase in the number of affected pairs when a smaller effective radius $R/3$ is used. One shortcoming of the method, as implemented here, is that the input $f_2(0)$ is not quite 2 for $\lambda=1$ but rather $(1+\lambda)(1+\alpha\lambda)\approx 1.6$. This could be solved by starting off with an input λ somewhat above unity.

The second algorithm, denoted BE_{23}, is a modification of the BE_3 form intended to give $C_2(0) = 1+\lambda$. The ansatz is

$$f_2(Q) = \left\{1 + \lambda \exp(-Q^2R^2)\right\} \times \left\{1 + \alpha\lambda \exp(-Q^2R^2/9) \times \left(1 - \exp(-Q^2R^2/4)\right)\right\} , \quad (8)$$

which is again applied only to identical pairs. The combination $\exp(-Q^2R^2/9)\left(1 - \exp(-Q^2R^2/4)\right)$ can be viewed as a Gaussian, smeared-out representation of the first dip of the cos function. As a technical trick, the $\delta\mathbf{r}_i^j$ are found as in the BE_3 algorithm and thereafter scaled down by the $1 - \exp(-Q^2R^2/4)$ factor. (This procedure does not quite reproduce the formalism of (3), but comes sufficiently close for our purpose, given that the ansatz form in itself is somewhat arbitarary.) One should note that, even with the above improvement relative to the BE_3 scheme, the observable two-particle correlation is lower at small Q than in the BE_0 algorithm, so some further tuning of λ could be required. In this scheme, $\langle\alpha\rangle \approx -0.25$.

It is interesting to note that the 'tuning' of α for energy conservation could have its analogue in global event weight algorithms. As we have noted above, a global weight would have to have an average value of unity to agree with theory and data, and this could be achieved (brute-force) by tuning the form of the weight expression appropriately. While our α is tuned event by event, the corresponding shape parameter(s) in global weight schemes would be tuned separately for each partonic configuration. To the extent that global weights start out close to an average of unity, the required tuning would be rather modest.

In the other two schemes, the original form of $f_2(Q)$ is retained, and the energy is instead conserved by picking another pair of particles that are shifted apart appropriately. That is, for each pair of identical particles i and j, a pair of non-identical particles, k and l, neither identical to i or j, is found in the neighborhood of i and j. For each shift $\delta\mathbf{p}_i^j$, a corresponding $\delta\mathbf{r}_k^l$ is found so that the total energy and momentum in the i, j, k, l system is conserved. However, the actual momentum shift of a particle is formed as the composant of many contributions, so the above pair compensation mechanism is not perfect. The mismatch is reflected in a nonunit value α used to rescale the $\delta\mathbf{r}_k^l$ terms.

The k, l pair should be the particles 'closest' to the pair affected by the BE shift, in the spirit of local energy conservation. One option would here have been to 'look behind the scenes' and use information on the order of production along the string. However, once decays of short-lived particles are included, such an approach would still need arbitrary further rules. We therefore stay with the simplifying principle of only using the produced particles.

Looking at $\mathrm{W}^+\mathrm{W}^-$ events and a pair i, j with both particles from the same W, it is not obvious whether the pair k, l should also be selected only from this W or if all possible pairs should be considered. Below we have chosen the latter as default behaviour, but the former alternative is also studied.

One obvious measure of closeness is small invariant mass. A first choice would then be to pick the combination that minimizes the invariant mass m_{ijkl} of all four particles. However, such a procedure does not reproduce the input $f_2(Q)$ shape very well: both the peak height and peak width are significantly reduced, compared with what happens in the BE_0 algorithm. The main reason is that either of k or l may have particles identical to itself in its local neighbourhood. The momentum compensation shift of k is at random, more or less, and therefore tends to smear the BE signal that could be introduced relative to k's identical partner. Note that, if k and its partner are very close in Q to start with, the relative change δQ required to produce a significant BE effect is very small, approximately $\delta Q \propto Q$. The momentum compensation shift on k can therefore easily become larger than the BE shift proper.

It is therefore necessary to disfavour momentum compensation shifts that break up close identical pairs. One alternative would have been to share the momentum conservation shifts suitably inside such pairs. We have taken a simpler course, by introducing a suppression factor $1 - \exp(-Q_k^2R^2)$ for particle k, where Q_k is the Q value between k and its nearest identical partner. The form is fixed such that a $Q_k = 0$ is forbidden and then the rise matches the shape of the BE distribution itself. Specifically, in the third algorithm, BE_m, the pair k, l is chosen so that the measure

$$W_{ijkl} = \frac{(1 - \exp(-Q_k^2R^2))(1 - \exp(-Q_l^2R^2))}{m_{ijkl}^2} \quad (9)$$

is maximized. The average α value required to rescale for the effect of multiple shifts is 0.73, i.e. somewhat below unity.

The BE_λ algorithm is inspired by the so-called λ measure [18] (not the be confused with the λ parameter of $f_2(Q)$). It corresponds to a string length in the Lund string fragmentation framework. It can be shown that partons in a string are colour-connected in a way that tends to minimize this measure. The same is true for the ordering of the produced hadrons, although with large fluctuations. As above, having identical particles nearby to k, l gives undesirable side effects. Therefore the selection is made so that

$$W_{ijkl} = \frac{(1 - \exp(-Q_k^2R^2))(1 - \exp(-Q_l^2R^2))}{\min_{(12 \text{ permutations})}(m_{ij}m_{jk}m_{kl}, m_{ij}m_{jl}m_{lk}, \ldots)} \quad (10)$$

is maximized. The denominator is intended to correspond to $\exp\lambda$. For cases where particles i and j comes from the same string, this would favour compensating the energy using particles that are close by and in the same string. This is thus close in spirit to some of the global approaches [32,11]. We find $\langle\alpha\rangle \approx 0.73$, as above.

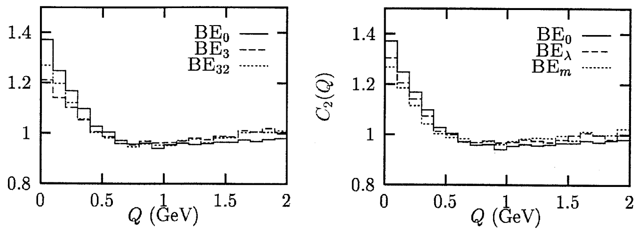

Fig. 2. The BE enhancement w.r.t. the no-BE case of the like-signed $\pi\pi$ correlation function in Z^0 decays as a function of Q

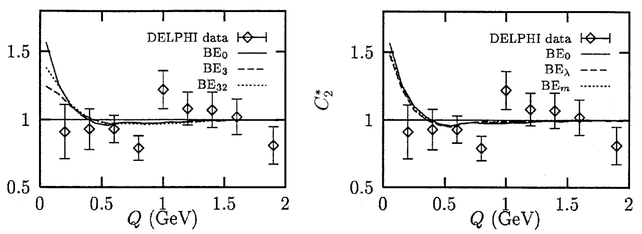

Fig. 3. The ratio between the like-signed and unlike-signed $\pi\pi$ correlation function as a function of Q, restricted to pairs of particles stemming from different W bosons in $e^+e^- \to W^+W^-$ events at LEP 2 according to the procedure in [13]

4 Results

Armed with these new algorithms we can now proceed to estimate BE effects. First consider the two-particle correlation function for like-sign π pairs from Z^0 decays normalized to a no-BE world, Fig. 2. All four algorithms were used with the same $\lambda = 1$ and $R = 0.5$ fm, but still show noticable differences. The enhancement at small Q is smallest in the BE_3 algorithm, as should be expected from the simpleminded way in which we picked the form of the energy-compensating below-unity extra factor. In all cases we expect that the parameters λ and R can be adjusted to reproduce experimental data.

In the introduction we mentioned the result presented by the DELPHI collaboration [13], where they found no trace of BE correlations among particles from different W bosons in fully hadronic $e^+e^- \to W^+W^-$ event. This was done by studying the ratio

$$C_2^*(Q) = \frac{N^{\pm\pm}_{WW\to 4j}(Q) - 2N^{\pm\pm}_{WW\to 2j\ell\nu}(Q)}{N^{+-}_{WW\to 4j}(Q) - 2N^{+-}_{WW\to 2j\ell\nu}(Q)} \ . \qquad (11)$$

Thus the numerator is the distribution in Q of like-sign pairs from fully hadronic events, subtracted with twice the distribution from semi-leptonic events. In the limit that the two W's hadronize completely independently, this difference is then made up of pairs where one particle comes from each W. The denominator is the same for unlike-signed pairs, which here should provide a good reference sample: with one particle of the pair from each W there is not going to be any of the resonance peaks that appear for distributions inside a W. In Fig. 3 we compare this result with the prediction from our algorithms, using the same parameters as in Fig. 2. Contrary to the data our models predict a clear BE enhancement for Q close to zero. The experimental statistics (only 24 hadronic and 25 semi-leptonic events were used) is not large enough to actually rule out the models. During the lifetime of LEP 2, the statistics is expected to grow by a factor 50, by which time it certainly would be possible to rule out our models, should the absence of BE enhancement in the data persist.

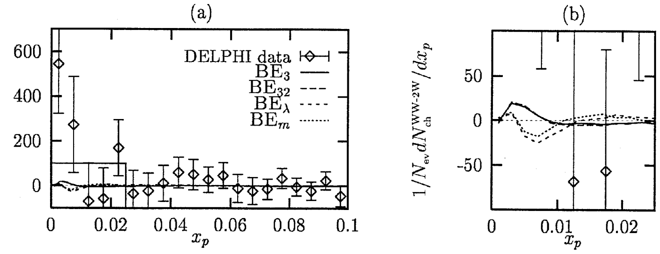

Fig. 4. The difference in x_p distributions of hadronic W decays between fully hadronic and semi-leptonic $e^+e^- \to W^+W^-$ events. Data from [34]. **b** is a detail view of the region close to the origin in **a**

Comparing fully hadronic and semi-leptonic $e^+e^- \to W^+W^-$ events, one can also find other observables which may be influenced by BE, and other interconnection effects between the two W systems. In [34] DELPHI found a hint of enhancement in charged multiplicity of fully hadronic events as compared with twice the multiplicity of isolated W decays. Also they found an indication of an increase in the multiplicity for small momentum fractions $x_p = 2p_h/E_{CM}$ of the hadrons. Both of these results could be signals for BE 'cross-talk' between the W's, but at present the errors are much too large to allow for any conclusions.

In Fig. 4 we present the predictions for the difference in x_p distributions between $e^+e^- \to W^+W^-$ events with and without cross-talk for our different algorithms. We see a small effect in the multiplicity at small x_p. However since the local reweighting scenario conserves the total multiplicity, any enhancement must be compensated, and this is also predominantly done at small x_p. The difference between the hadronic and leptonic W decays is therefore the result of a subtraction between almost equally large numbers, thereby emphasizing details of the algorithms.

Thus, in all our algorithms, the energy-momentum conservation procedure reduces the effect of BE enhancement in the x_p spectra at small x_p. Indeed the enhancements are in all cases much smaller than is indicated by data. Given the large experimental errors we do not take this seriously, in particular since L3 and OPAL do not confirm the DELPHI observation [42]. However, should the signal in [34] survive an increase in statistics, it would not necessarily rule out our local reweighting approach as such, but need only indicate that we still have a problem with the approach to the energy-momentum conservation issue. A difference at small x_p could also be caused by other physics mechanisms, such as colour rearrangement [16,17].

We now proceed to estimate the BE-induced shift in the measured W mass m_W^{4j}. Since our algorithms preserve the notion that each particle belongs to a given W, it is

Table 1. Shifts in MeV of the measured mass m_{W}^{4j} for different models and different mass reconstruction methods. The top number in each column indicates the statistical error for a simulated sample of 4×10^5 events. The event samples were generated at 170 GeV center of mass energy (except the last two rows which were generated at 190 GeV) and the fits were restricted to $78.25 < m_{\mathrm{W}}^{4j} < 82.25$ GeV (except the row $\mathrm{BE}_m^{\mathrm{peak}}$ uses $79.2 < m_{\mathrm{W}}^{4j} < 81.3$ GeV)

model	$\langle \delta m_{\mathrm{W}} \rangle$	$\delta \langle m_{\mathrm{W}}^{4j0} \rangle$	$\delta \langle m_{\mathrm{W}}^{4j\mathrm{A}} \rangle$	$\delta \langle m_{\mathrm{W}}^{4j\mathrm{B}} \rangle$	$\delta \langle m_{\mathrm{W}}^{4j\mathrm{C}} \rangle$
	±1	±4	±8	±8	±8
(170 GeV)					
BE_0	130				
BE_3	−8	−6	−4	1	−6
BE_{32}	−9	−8	−3	−5	−2
BE_λ	38	38	16	15	12
BE_m	75	69	15	13	14
BE'_m	59	50	2	8	−5
BE''_m	102	93	26	25	23
BE_m^L	60	44	17	19	11
$\mathrm{BE}_m^{\mathrm{peak}}$	75	70	18	13	16
(190 GeV)					
BE_m	183	191	23	25	14
BE'_m	127	114	−8	−14	−8

easy to obtain a shift in each event by simply calculating the invariant mass of the decay products of each W before and after the BE algorithm. The average shift is presented in Table 1 in the column denoted $\langle \delta m_{\mathrm{W}}^{4j} \rangle$. It is clear that the shifts obtained with the new algorithms are smaller than our previous result in [6][1]. This is to be expected as the energy is conserved locally by pushing pairs of particles away from each other, counteracting the BE-induced shift. Especially in the BE_3 and BE_{32} schemes, the opposing shift is calculated between the same particles as are affected by the BE shifts, and it is not surprising that the total shift in the W mass is close to zero. Put another way, we have previously argued, on physics grounds, that weights above unity naturally leads to a positive W mass shift, and it follows in the same spirit that weights below unity gives a negative W mass shift. In the BE_3 and BE_{32} schemes, weights above and below unity are tuned in such a way that their net effect is expected to cancel, exactly for energy and approximately for the W mass.

In Table 1 we also present the result for some variations of the BE_m scheme. BE'_m is explained below. For BE''_m, if a pair of identical bosons come from the same W, only pairs of particles from this W are considered for the energy compensating shift. In BE_m^L, the shifts $\delta \mathbf{p}_i^j$ and $\delta \mathbf{r}_i^j$ are calculated in the center of mass system of each pair instead of in the lab system. In both these cases the changes are moderate and remind us that there are uncertainties due to the details in the implementation.

It has been noted that a real measurement of m_{W}^{4j} would mostly be sensitive to the peak position of the mass distribution, and in [9] it was found that the small BE-induced shift in $\langle \delta m_{\mathrm{W}}^{4j} \rangle$ mostly stem from the tails of the distribution. The BE shift thus almost disappears if the mass is obtained from a fit to a relativistic Breit–Wigner (plus background). Doing the same with our algorithms we find no significant decrease of the BE shift, however, as seen in the column denoted $\delta \langle m_{\mathrm{W}}^{4j0} \rangle$ in Table 1. It is possible this partly comes from the difference between models with global weights and those without. Specifically, if the average value of the global weight has a nontrivial energy dependence, then the weighting procedure would skew the wings. However, this is just a guess, and further studies are required to settle the issue.

It is clear that the mass shift in our algorithms would mostly come from the softest particles in the events. These are also the ones that are most difficult to associate to one or the other of W^+ and W^-. To achieve a more experimental-like situation we therefore ignore what the generator tells us about the origin of each final-state particle and instead perform a jet clustering in the same way as in [16]. Three different strategies are studied for associating jets with either W boson, denoted A, B, and C in Table 1. In all cases the LUCLUS jet clustering algorithm [7] is used to reconstruct exactly four jets. These are then paired together to represent a W^+ and a W^-. In each event the combination $(j_1 j_2)(j_3 j_4)$ is chosen which minimises $|m_{j_1 j_2} - 80| + |m_{j_3 j_4} - 80|$ (A) or $|m_{j_1 j_2} - 80 + m_{j_3 j_4} - 80|$ (C) or maximizes the angles between the jets $\theta_{j_1 j_2} + \theta_{j_3 j_4}$ (B). The reconstructed mass distribution is thereafter again fitted to extract a peak position.

In all cases the BE-induced shift is reduced. It seems that the BE-shifts increases the likelihood that soft particles become misassigned in such a way that the momenta of the W's are increased. (We remind that, by energy conservation, an increased W momentum corresponds to a decreased W mass.) To see how this can come about, assume that the four jets of an event separate into one W^+ and one W^- hemisphere, i.e that the two jets of the W^+ (W^-) have a positive longitudinal momentum with respect to the W^+ (W^-) direction of motion. Stray particles in the 'wrong' hemisphere would then have a large likelihood of being misassigned. Such a misassignment removes particles with momentum opposite to the motion of the W itself and adds them to the other W, thus increasing the reconstructed momentum of both. Since our implementation of BE effects tends to enhance particle production in the central region of the event and particularly the migration of particles in the direction of the other W, we would then expect an effect of the observed sign. When the jets of a W are not in the same hemisphere, the effects of misassignments could more easily go either way, so the influence on the W mass should be reduced. To quantify effects, consider events aligned with the W^+ along the $+z$ axis and then require $\delta p_z = |p_{zq_1} - p_{z\bar{q}_2}| + |p_{zq_3} - p_{z\bar{q}_4}| < E_{\mathrm{CM}}/2$, using generator information about the z-components of the initial quarks from the W decays ($\mathrm{W}^+ \to q_1 \bar{q}_2$, $\mathrm{W}^- \to q_3 \bar{q}_4$). Using a simple cut at $p_z = 0$ we can get an estimate of the BE-induced misassignment effects by studying the difference in p_z distribution of particles from one W with and

[1] The corresponding value in [6] is somewhat lower due to a minor error in the averaging procedure

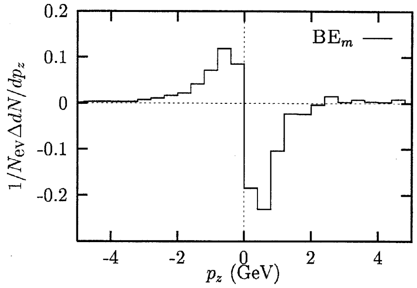

Fig. 5. The difference between the p_z distribution with and without BE correlations between the W's according to the BE$_m$ algorithm, for events with $|p_{zq1} - p_{zq2}| + |p_{zq3} - p_{zq4}| < E_{\mathrm{cm}}/2$. p_z for a particle is the momentum component along the direction of the W from which it was produced

Table 2. Shifts in the W mass peak position due to reconstruction and BE effects for different topologies. Low means $\delta p_z < E_{\mathrm{CM}} * 0.4$, high means $\delta p_z > E_{\mathrm{CM}} * 0.6$. $\Delta\langle m_{\mathrm{W}}^{4j\mathrm{A}}\rangle$ is the shift in the peak position due to the reconstruction, while $\delta\langle m_{\mathrm{W}}^{4j0}\rangle$ and $\delta\langle m_{\mathrm{W}}^{4j\mathrm{A}}\rangle$ are defined as for Table 1. The statistical error is everywhere around 10 MeV. Note the relation $\delta\langle m_{\mathrm{W}}^{4j\mathrm{A}}\rangle = \delta\langle m_{\mathrm{W}}^{4j0}\rangle + \Delta\langle m_{\mathrm{W}}^{4j\mathrm{A}}\rangle(\mathrm{BE} - \mathrm{no\ BE})$, i.e. the observable W mass shift by BE effects is the sum of the theoretical mass shift for 'correct' assignment of particles and the mass shift by 'erroneous' particle assignments when moving from the no-BE to the BE world

shift	low	medium	high
$\Delta\langle m_{\mathrm{W}}^{4j\mathrm{A}}\rangle$ no BE	-62	+175	+189
$\delta\langle m_{\mathrm{W}}^{4j0}\rangle$ BE$_m$	+88	+64	+52
$\Delta\langle m_{\mathrm{W}}^{4j\mathrm{A}}\rangle$ BE$_m$	-137	+139	+134
$\delta\langle m_{\mathrm{W}}^{4j\mathrm{A}}\rangle$ BE$_m$	+13	+28	-3

without BE-cross-talk. The result is shown in Fig. 5 for the BE$_m$ algorithm and we see that the misassignment is indeed increased. Integrating the curve in Fig. 5 we find an average increase in the W momentum of the order of 100 MeV, which would correspond to a shift in the reconstructed W mass of about −40 MeV. Note that this shift is negative, so the statement in our previous publication [6] that BE effects necessarily would increase the measured mass in not quite true. Note also that one could imagine that BE effects in this way could affect the measured mass even if the actual W masses are unaffected. For the BE$_{32}$ algorithm, however, the BE-induced misassignment effects are much smaller and we see no effect for the A, B and C reconstruction in Table 1.

Looking more closely at the effects of reconstruction method A, we see in Table 2 that without any BE cross-talk, the measured W mass is affected differently for different event topologies (again using δp_z above as a topology measure). For small δp_z the mass is shifted downwards, while for larger δp_z, the shift is positive. In Table 2 we also see that the *direct* BE shift is positive everywhere, although largest at small δp_z. But with BE cross-talk, the reconstruction effects are also changed, and the reconstructed mass is lowered everywhere as compared with the case of no cross-talk. At small δp_z, where the direct

Table 3. The fitted width for different models and different mass reconstruction methods. Notation as in Table 1. Also shown is σ_{BE}, the Gaussian width of the true BE-induced mass shift

model	σ_{BE}	$\delta\langle\Gamma_{\mathrm{W}}^{4j0}\rangle$ ± 10	$\delta\langle\Gamma_{\mathrm{W}}^{4j\mathrm{A}}\rangle$ ± 31	$\delta\langle\Gamma_{\mathrm{W}}^{4j\mathrm{B}}\rangle$ ± 34	$\delta\langle\Gamma_{\mathrm{W}}^{4j\mathrm{C}}\rangle$ ± 28
BE$_3$	36	6	44	49	49
BE$_{32}$	47	8	28	27	39
BE$_\lambda$	250	80	48	36	29
BE$_m$	190	34	44	39	42
BE$'_m$	180	31	66	76	70
BE$''_m$	140	6	54	51	44
BE^L_m	170	29	28	24	30
BE$^{\mathrm{peak}}_m$	190	56	48	49	28

BE shift is largest, the additional negative shift due to BE-induced reconstruction effects is also larger, and everywhere the direct BE shift is more or less compensated by BE effects in the reconstruction.

Above we noted the increase in fitted W width in some global weight models. Also in our models is the width increased by BE effects, Table 3. The order of the width increase is 40 MeV, i.e. comparable with what is found in the global models. In retrospect, a broadening of the W peak is a not unnatural consequence of the fluctuations in the BE-induced W mass shifts. That is, till now we have discussed the shift of the *average* W mass in a large event sample. The shift in each *individual* W mass is much larger, typically 200 MeV, cf. Table 3. This variability is rather weakly correlated with the W mass itself, but is instead mainly given by the W decay angles and fluctuations in the fragmentation process. The observable W width is therefore increased in relation to the width of the BE mass shift distribution. A crude addition in quadrature gives the right order of magnitude of the effects, $\delta\langle\Gamma_{\mathrm{W}}\rangle \sim \Gamma_{\mathrm{BE}}^2/2\langle\Gamma_{\mathrm{W}}\rangle \sim 2\sigma_{\mathrm{BE}}^2/\langle\Gamma_{\mathrm{W}}\rangle$. One should note, however, that the error on the W width determination is rather large, so it is doubtful whether a 40 MeV increase in the W width will be observable at LEP 2. Specifically, our models give only a very modest drop of peak height, Fig. 6, and the total cross section in the central peak is essentially unchanged. This should be contrasted with the model of Fiałkowski and Wit, where there is a significant increase of the low-mass tail, beyond the range of the W peak fit, and a corresponding drop of the peak value. Whereas thus an increase of the W width seems to be a common phenomenon in many models, the difference is whether this is mainly a broadening of the central mass peak or also has significant implications for the wings.

In [6] we noted that the shift in m_{W}^{4j} increases with the center of mass energy, and explained why this is a natural behaviour. This is still true e.g. for the BE$_m$ model, as seen in Fig. 7. However, the argumentation is based on the assumption that the fragmentation regions of the W$^+$ and the W$^-$ do overlap significantly, as is the case over the LEP 2 energy range. At very high energies the shift should go away, since here the W's decay only after they have travelled well apart. The separation of the decay ver-

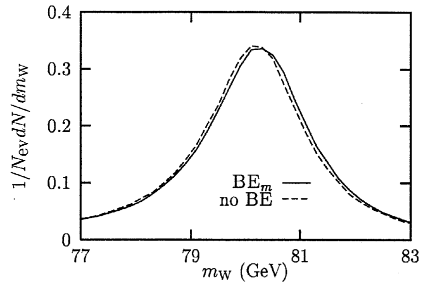

Fig. 6. Shape of the W mass peak with and without BE included according to model BE_m

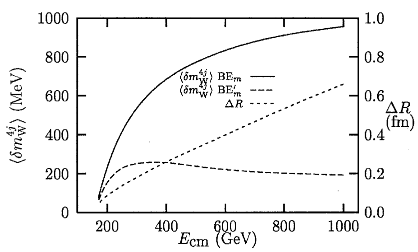

Fig. 7. The average shift in MeV of the measured mass m_{W}^{4j} for the BE_m and BE'_m models as a function of the $\mathrm{e}^+\mathrm{e}^-$ center of mass energy (in GeV). Also shown is the average separation ΔR in fm between the decay vertices of W^+ and W^-

tices can be taken into account, approximately, by using a modified radius in the $f_2(Q)$ function in (3) when calculating shifts for particles from different W bosons (but not from the same W). Specifically, the procedure described in [16] is used to generate the distance ΔR between the decay vertices of the two W's, based on a Monte Carlo sampling of the expected W decay distribution as a function of the W mass. We then define a modified version of BE_m, denoted BE'_m, with

$$f_2^{+-}(Q) = 1 + \lambda \exp(Q^2(R + \Delta R)^2) \qquad (12)$$

for pairs from different W's. In Fig. 7 it is seen that the separation does indeed lower the shift, but the shift still rises with the center of mass energy until around 400 GeV, whereafter it slowly decreases. The effect of the experimental reconstruction procedures can be seen from Table 1, where the BE_m model shows the expected increase, while BE'_m remains close to zero and possibly is decreasing towards more negative values. The net uncertainty therefore indeed does seem to increase over the LEP 2 energy range.

5 Conclusions

This paper has two objectives: to take a critical look at the modelling of BE effects, especially for its impact on the W mass, and to develop improved versions of local weight algorithms.

Today the 'global weight' approach to the BE phenomenon dominates. However, many global weight schemes have basic weaknesses, in areas such as the theoretical one of factorization or the experimental ones of comparisons with Z^0 total and partial widths, cross sections, jet rates, and so on. Furthermore, one can easily see ways to construct global weight models that could give misleading results, e.g. if the average BE weight has a non-trivial dependence on the mass of each W or on the jet topology of the W decays. In general, the arbitrariness of the weight rescaling schemes probably is the limiting factor when trying to extract reliable predictions out of several current global weight algorithms. Even when factorization is respected, there is no unique recipe for how BE effects could couple the two W hadronization processes.

Therefore we do not consider the matter settled. The local weight approach is certainly not free of objections, but it does address and solve some of the basic issues that the pure global weight approach does not. However, just as there exist a multitude of mutually contradictory global models in the literature, one can construct many kinds of local models. In this paper we have come up with four main alternatives to the scheme in [6]. For technical simplicity, all four are based on the same kind of momentum shifting strategy as in the original one, but they are still sufficiently different to probe a wide space of local weight models. The models are in this paper applied to the topical issue of the W mass, but clearly can be used also for Z^0 physics and other studies. Therefore, should the experimental verdict be that no BE effects connect the two W's, the algorithms we have proposed here could still be used to explore other aspects of the BE phenomenon. Should an effect be found, on the other hand, it would be even more interesting to understand whether the algorithms can be discriminated by more detailed comparisons with LEP 1 data.

In our original paper [6] we stated that the model studied there was likely to give an estimate of the maximal possible effects, with the real ones some unknown fraction thereof. Indeed, the models studied here at most reach three quarters of the original W mass shift, and range down to essentially zero mass shift. This is based on untuned models, however, and we expect that a careful tuning to Z^0 data would bring up the numbers somewhat. Global weight models cluster around zero. There are exceptions that show some shift, but none anywhere near as big as our original scenario. Furthermore, with the new models we now have the possibility to study the impact of the experimental procedures used to extract a W mass. Unlike the results of [9], a fit to the peak position of the W Breit-Wigner does not significantly reduce the theoretical mass shift in our models. Instead a reduction occurs by another mechanism: the shift of the momenta of particles belonging to one W in the direction of the other W. So long as these particles are bookkept with their original W, it is precisely this mechanism that reduces the W momenta and hence increases the W masses in the first place. When particles are shifted so far that they tend to be assigned to the 'wrong' W, however, the reconstructed momentum of each W can instead increase and the W mass shift is thereby reduced.

In the end, we therefore remain with W mass shifts up to at most 30 MeV at 170 GeV. These models still have to be retuned somewhat, cf. Fig. 2, and the uncertainty would increase with energy, but something like 50 MeV seems to be a safe upper limit over the LEP 2 energy range. All the numbers here refer to our attempts at reproducing a sensible experimental procedure. As we have seen, however, the BE phenomenon does involve low-momentum particle and contains nontrivial dependences on the event topology, so the only realistic numbers are those that are obtained by the experimental collaborations, with their selection cuts and within their acceptance. Disregarding such issues, it would be tempting to take some kind of average of the different model studies, ours and those of others, and claim that the uncertainty on the W mass from BE effects is even smaller, maybe not more than 10–15 MeV. However, nature is not a democratic compromize between ten models. There exists *one* correct description of BE effects and, if we are honest, we have to admit that all the models we use are likely to be flawed with respect to this truth. Therefore an estimate of the uncertainty had better be based on the most 'pessimistic' scenario that is not in blatant disagreement with existing data.

This does not mean prospects are hopeless. The DELPHI [13] and ALEPH [14] studies point the way to constraining the amount of cross-talk occuring between the W^+ and W^- hadronic systems, once the statistics is improved. An observation of no cross-talk would certainly settle the issue, in the sense that we (at least currently) do not know of any way to construct a BE model that would give a $C_2^*(Q) \equiv 1$ ((11)) and still induce a W mass shift. However, note that the converse does not hold: models with similar nonunity $C_2^*(Q)$ shapes may disagree on the W mass shift value. What can be said, however, is that the closer $C_2^*(Q)$ is constrained to unity, the smaller the maximum imaginable W mass shift.

While clearly the observation of BE effects spanning the two W's would be very exciting, also a null result would be very interesting and in need of an explanation. (How do two hadronizing systems, that clearly overlap in space and time, manage not to feel each other?) Continued BE studies therefore are well worth the effort.

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